A Modeling and Experimental Study of the Influence of Twist on the Mechanical Properties of High-Performance Fiber Yarns

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ABSTRACT: Little data exist on how twist changes the properties of high-performance continuous fiber yarns. For this reason, a study was conducted to determine the influence of twist on the strength and stiffness of a variety of high-performance continuous polymeric fiber yarns. The materials investigated include Kevlar 29[®], Kevlar 49[®], Kevlar 149[®], Vectran HS[®], Spectra 900[®], and Technora[®]. Mechanical property tests demonstrated that the initial modulus of a yarn monotonically decreases with increasing twist. A model based on composite theory was developed to elucidate the decrease in the modulus as a function of both the degree of twist and the elastic constants of the fibers. The modulus values predicted by the model have good agreement with those measured by experiment. The radial shear modulus of the fiber, which is difficult to measure, can be derived from the regression parameter of experimental data by the use of the model. Such information should be useful for some specialized applications of fibers, for example, fiber-reinforced composites. The experimental results show that the strength of these yarns can be improved by a slight twist. A high degree of twist damages the fibers and reduces the tensile strength of the yarn. The elongation to break of the yarns monotonically increases with the degree of twist. © 2000 John Wiley & Sons, Inc. J Appl Polym Sci 77: 1938-1949, 2000

Key words: twisted yarn; Young's modulus; modeling; high-performance fibers

INTRODUCTION

Since the 1970s, considerable effort in the fiber industry has been dedicated to finding new polymeric fibers with high performance for potential use in applications such as ropes and cables, fiber-reinforced composites, ballistic vests, and gaskets. Kevlar[®],¹ Vectran[®],² Technora[®],³ and Spectra^{®4,5} are some excellent examples of highperformance polymeric fibers. There have been

Contract grant sponsor: Materials Research Science and Engineering Center; contract grant number: DMR 9809635. Journal of Applied Polymer Science, Vol. 77, 1938–1949 (2000) © 2000 John Wiley & Sons, Inc. several investigations of the properties and the structure-property relationships of these high-performance fibers.⁶⁻¹⁰ These fibers show high anisotropy^{11,12} when compared to conventional fibers or other bulk materials. This article aimed to illustrate the importance of material anisotropy on some macroscopic properties by studying the properties of twisted yarn.

Special geometric effects are introduced into fiber yarns in many textile applications. Twist has long been known to improve the strength of short-fiber staple materials and few recognize that many threads are composed of short fibers that are held together by twist. Strands of twisted fibers have been used to control flexibility in copper and steel cable systems. Twist is also used to give continuous yarns integrity and force the as-

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sembly of single fibers to behave as a single unit. For conventional fibers such as nylon and polyester, only a small amount of twist is used because twist damages critical yarn properties. However, for some high-performance fibers, twist affects the yarn properties such as modulus, strength, and elongation to break in a more complicated way. There are few reports describing the influence of twist on the properties of high-performance polymeric fiber yarns.^{11–13} One of the purposes of this article was to illustrate and model the influence of twist on high-performance fiber yarns.

As early as 1907, Gegauff¹⁴ proposed a simple analysis to correlate the twist angle of a yarn with the yarn modulus. Gegauff's derivation is given below and forms an excellent basis to expand this type of model.

The geometry of a twisted yarn is shown in the following:



where (a) is an ideal geometry of a twisted yarn; the solid curve line, a single filament; R, the radius of the yarn; r, the radial position of a single filament; h, the yarn length for one turn; δh , the increase in length h under stretching; and θ , the angle between the path of the single filament at a radius of r and the yarn axis, and for the single filament at the radius of R, the angle is α , and (b) is a flattened projection of a single filament; l, the length of a single filament at a radius of r; and δl , the corresponding filament elongation.

Gegauff's Classic Model of Twisted Yarn Mechanics

1. Relation of single filament extension to yarn extension.

Let the yarn extension $= \varepsilon_1 = \delta h/h$; then, the fiber path length $= l = h/\cos \theta$ and the fiber displacement $= \delta l = \delta h \times \cos \theta$. Consequently,

the filament extension

$$= \varepsilon_{\rm f} = \frac{\delta l}{l} = \cos^2 \theta \times \frac{\delta h}{h}$$

Thus,

the yarn extension $= \varepsilon_1 = \varepsilon_{\ell}/\cos^2\theta$

2. Axial tensile force.

The stress along a filament in the yarn is

$$\sigma_f = E_f \times \varepsilon_f$$

and an equivalent area perpendicular to the filament axis = $2\pi r dr \cos \theta$.

Then, at a position of radius r in the yarn, the component of tension parallel to the yarn axis resulting from the filament stress becomes $= E_f \varepsilon_f (2\pi r dr \cos \theta) \cos \theta = E_f 2\pi r dr \varepsilon_1 \cos^4 \theta$. Therefore, total yarn tension $= \int_0^R E_f 2\pi r dr \varepsilon_1 \cos^4 \theta$ $= \pi R^2 \varepsilon_1 E_f \cos^2 \alpha$ Thus, yarn modulus $= \hat{E} = E_f \cos^2 \alpha$

However, the measured yarn modulus always shows a stronger dependency on the degree of twist than what the model indicated. Platt¹⁵ followed this approach and incorporated the effect of lateral contraction and large extension. Hearle¹⁶ added the influence of transverse forces and neglected lateral contraction. Treloar and Riding¹⁷ adopted an energy method instead of performing a stress analysis and considered the migration of a single filament in a twisted yarn. Freeston and Platt¹⁸ further considered the effect of the strain rate on the stress-strain behavior. White et al.¹⁹ performed a continuum mechanics analysis including not only transverse forces but also interfilament friction. They derived a more complicated equation describing the influence of twist on the yarn modulus:

Yarn modulus $= \hat{E} = E_f$

$$imes \left(rac{1}{4} + rac{9}{4} \, {T}_{0} + rac{3{T}_{0}}{(1-{T}_{0})} \, {
m ln}{T}_{0}^{1/2}
ight)$$

in which $T_0 = \cos^2 \alpha$.

None of the models described above provides good agreement with the experimental data, especially when yarns made from high-performance fibers are considered. All these models imply that the change in the Young's modulus of a material depends only upon the angle between the material axis and the stress direction regardless of other material properties. The first drawback of these models lays in the fact that, for an isotropic material, the modulus does not change with the relative direction of the stress with the material axis. Second, as shown later in this article, yarns made from different fiber materials behave differently at the same twist angle, which is not contained in any of the previous models. Therefore, the omission of important material characteristics, such as anisotropy, yields models that are

Kevlar[®] (Dupont)

Vectran[®] (Hoechst Calaneses)

Technora® (Teijin)

Spectra[®] (Allied Signal)

Twisted yarns were made by anchoring one end of a fixed length of yarn and, while maintaining a slight tension, rotating the other end about the yarn direction a predefined rotation. The length of the yarn after twisting was recorded and the twist in these yarns was calculated according to ASTM D 1432-92. Several terms were used to characterize the degree of twist in a yarn. Twist in turns per inch (tpi) was directly counted. A term called the twist multiplier (TM) is related to the yarn twist in tpi and yarn denier by the equation

$$\mathrm{TM} = rac{\mathrm{tpi} imes \sqrt{\mathrm{denier}}}{73}.$$

The surface twist angle is the angle between the filament at the most outer layer of a twisted yarn and the yarn axis. The linear density of the yarn was measured by a microbalance to determine the yarn denier, that is, the weight in grams of 9000 unsatisfactory. In this article, the composite theory is used to incorporate the anisotropy of the material into a new model that accurately describes the influence of twist on the yarn modulus for all polymeric fibers.

EXPERIMENTAL

Commercial fiber yarns of Kevlar 29[®], Kevlar 49[®], Kevlar 149[®] (DuPont, Richmond, VA, USA), Vectran HS[®] (Hoechst Celanese, Wilmington, DE, USA), Technora[®] (Teijin, Japan), and Spectra 900[®] (Allied Signal, Petersburg, VA, USA) were used in this study. The chemical structures of these fibers are illustrated below:



meters of yarn. The fiber density was measured using a density-gradient column. By knowing the denier and density, the cross-sectional area of the yarn was calculated.

Tensile testing of the twisted yarn was done using an Instron[®] Model 5564 testing machine according to ASTM D 2256-90. All the tests were performed at standard conditions of 21° C (1°C) and 65% (2%) relative humidity. Pneumatic yarn/cord grips were used for these tests, with the effective gauge length set at 150 mm and a crosshead speed of 10%/min. The initial modulus was calculated from the slope of the stress–strain curve in the strain range from 0.05 to 0.5%. Griping effects were corrected by adjusting the gauge length.

Notation

Given below is a list of the principal symbols and notation used in the text. To avoid confusion and complexity, all the symbols are defined here. Re-



Figure 1 Schematic illustration of the model of calculation of the twisted yarn modulus from the fiber properties.

ferring to Figure 1, different structural components—a single filament, a layer of flattened twisted yarn composed single filaments resembling a unidirectional composite, and a twisted yarn—are utilized in this article. The properties of a single filament are transformed first into on-axis properties and further into off-axis properties of the unidirectional composite.

For a single filament and twisted yarn, the symbols follow what was previously defined:

- E_{f} , ε_{f} , and σ_{f} characterize the behavior of a single filament.
- ε_1 and σ_1 characterize the behavior of a twisted yarn.
- $\hat{E}(\alpha)$ is the apparent Young's modulus of a twisted yarn with the surface twist angle of α .

For the layer of flattened twisted yarn, that is, the unidirectional composite, the on-axis constitutive equation can be expressed by the following matrix:

$$\begin{array}{c} \varepsilon_z \\ \varepsilon_y \\ \varepsilon_s \end{array} \end{pmatrix} = \left(\begin{array}{c} S_{zz} & S_{zy} & 0 \\ S_{yz} & S_{yy} & 0 \\ 0 & 0 & S_{ss} \end{array} \right) \left(\begin{array}{c} \sigma_z \\ \sigma_y \\ \sigma_s \end{array} \right) \\ = \left(\begin{array}{c} \frac{1}{E_z} & \frac{-\nu_{zy}}{E_y} & 0 \\ \frac{-\nu_{yz}}{E_z} & \frac{1}{E_y} & 0 \\ 0 & 0 & \frac{1}{E_s} \end{array} \right) \left(\begin{array}{c} \sigma_z \\ \sigma_y \\ \sigma_s \end{array} \right)$$

and the off-axis constitutive equation can be expressed by the following matrix:

$$egin{array}{l} \left(egin{array}{c} arepsilon_1 \ arepsilon_2 \ arepsilon_6 \$$

in which z and y are principal terms of the on-axis behavior; 1 and 2, principal terms of the off-axis behavior; s and 6, shear terms; ε_i , the strain; σ_i , the stress; S_{ij} and E_i , the compliance and engineering Young's modulus; and ν_{ij} , Poisson's ratio.

On-axis properties of a twisted yarn layer are the same as the properties of a single filament, $E_z = E_f$; $E_s = E_{rz}$; and $E_y = E_r$, in which E_{rz} is the shear modulus and E_r is the transverse radial modulus in a filament. θ , α , r, and R have the same meanings as previously defined in the ideal twist geometry and dominate the characteristic off-axis behavior of different yarn layers. T_0 is a function of the surface twist angle, $\cos^2\alpha$; d, the anisotropic ratio, E_z/E_s ; E_{zc} , E_{yc} , and E_{sc} , the onaxis Young's moduli and shear modulus of an unidirectional composite; and E_m and G_m , the Young's modulus and shear modulus of the matrix in a composite.

MODELS, RESULTS, AND DISCUSSION

The approach used to model how the varn modulus is influenced by the degree of twist is quite straightforward. A strategy was taken as shown schematically by Figure 1. The notation in the following discussion will follow that in Figure 1. The twisted yarn was first dissected into individual loading elements, which are a series of thinwall tubes. A tube is hypothetically cut and opened up to be a sheet structure with a preferred orientation different from that of the planar material axes. The stress-strain response of the plane was solved using orthotropic composite theory.²⁰ The responses of the individual elements were added via integration of the layered structures and this yields the stress-strain behavior of the yarn. In this way, the yarn modulus was derived from the single-filament properties.

1. Single-layer mechanics in a twisted yarn.

A thin-walled layer of the twisted yarn

was taken. The open-up structure of this layer resembles a unidirectional composite. At the radial position of r, the off-axis angle is θ , and at the radial position of R, the off-axis angle is α .

According to composite theory,²⁰ under uniaxial stress, a unidirectional composite responds as

$$\varepsilon_1 = S_{11}\sigma_1 = \frac{\sigma_1}{E_1}$$

Therefore, the off-axis modulus of a single layer $E_1 = 1/S_{11}$ and the off-axis stress

$$\sigma_1(\theta) = \frac{1}{S_{11}} \varepsilon_1 \tag{1}$$

2. Yarn mechanics by layer assembly.

Assuming no slippage between layers because of the integrity of a twisted yarn, each layer has the same extension $\varepsilon = \varepsilon_1$ under uniaxial stress. Also, assuming continuous layers along the radial direction in a twisted yarn, the axial force in a yarn is

$$F = \int \sigma_1(\theta) dA = \int_0^R \sigma_1(\theta) 2 \pi r dr \qquad (a)$$

If the yarn is treated as one body, the axial force can also be expressed as

$$F = \pi R^2 \hat{E} \varepsilon_1 \tag{b}$$

in which \hat{E} is the apparent yarn Young's modulus. Utilizing the ideal twist geometry,

$$\tan \theta = \frac{2\pi r}{h} \quad \text{and} \quad \tan \alpha = \frac{2\pi R}{h}$$

as well as the equality of eqs. (a) and (b) and the expression of $\sigma_1(\theta)$ by eq. (1), the apparent Young's modulus of a twisted yarn can be calculated as

$$\hat{E} = \frac{2}{\tan^2 \alpha} \int_0^\alpha \frac{1}{S_{11}(\theta)} \tan \theta \sec^2 \theta d\theta \qquad (2)$$

Exact Solution

According to composite theory,²⁰ the off-axis compliance of a unidirectional composite can be expressed as

$$S_{11} = m^4 S_{zz} + n^4 S_{yy} + m^2 n^2 S_{ss} + 2m^2 n^2 S_{zy} \quad (3)$$

where

$$S_{zz} = rac{1}{E_z}, \ S_{yy} = rac{1}{E_y}, \ S_{ss} = rac{1}{E_s}, \ S_{zy} = -rac{
u_{zy}}{E_y} = -rac{
u_{yz}}{E_z}$$

 $m = \cos \theta$, $n = \sin \theta$, and θ is the off-axis angle. The compliance S_{11} can be expressed by the moduli after rearrangement as

$$S_{11} = \frac{1}{E_y} + \left(\frac{1}{E_s} - \frac{2}{E_y} - \frac{2\nu_{yz}}{E_z}\right) \cos^2\theta + \left(\frac{1}{E_z} + \frac{1}{E_y} - \frac{1}{E_s} + \frac{2\nu_{yz}}{E_z}\right) \cos^4\theta \qquad (4)$$

Substitution of eq. (4) into eq. (2) yields after integration

Yarn modulus

$$\hat{E}(\alpha) = \frac{1}{\tan^2 \alpha} \left[\frac{b}{2c^2} \ln \frac{(a+b+c)T_0^2}{(aT_0^2+bT_0+c)} - \frac{T_0-1}{cT_0} + \frac{(b^2-2ac)}{2c^2\sqrt{b^2-4ac}} \ln \frac{(2a+b-\sqrt{b^2-4ac})}{(2aT_0+b-\sqrt{b^2-4ac})} \times (2a+b+\sqrt{b^2-4ac}) + (2a+b+\sqrt{b^2-4ac}) \right]$$
(5)

in which

$$egin{aligned} T_{0} &= & \cos^{2}lpha, \ lpha &= rac{1}{E_{z}} + rac{1}{E_{y}} - rac{1}{E_{s}} + rac{2
u_{yz}}{E_{z}}, \ b &= rac{1}{E_{s}} - rac{2}{E_{y}} - rac{2
u_{yz}}{E_{z}}, \ c &= rac{1}{E_{y}} \end{aligned}$$

Approximation

To reduce the rather complicated eq. (5) to a simpler form, several approximations were used. Since it is generally true that a high degree of twist damages the material properties, only low twist angles are usually used in real applications. The approximations given below are valid only for this condition. Another factor used to simplify the above expression is recognition of the fact that for high-performance fibers the ratio of the longitudinal modulus to the transverse modulus is usually greater than 10.

In case of a low off-axis angle ($\alpha < 15^{\circ}$),

$$m^2 \gg n^2 (n^2/m^2 < 7\%)$$

Taking a range of $-0.5 \le \nu_{yz} < 0.5$, which was found reasonable for PPTA fiber¹² and assumed for other fibers investigated, then

$$m^4 S_{zz} + 2m^2 n^2 S_{zy} = m^2 / E_z (m^2 - 2v_{yz}n^2) \approx m^4 S_{zz}$$

For , $S_{yy} \sim S_{ss}$,
 $n^4 S_{yy} + m^2 n^2 S_{ss} = n^2 (n^2 S_{yy} + m^2 S_{ss}) \approx m^2 n^2 S_{ss}$

Then,

$$S_{11} = m^4 S_{zz} + m^2 n^2 S_{ss} \tag{6}$$

Now, the anisotropic ratio is defined as $d = E_z/E_s$, which is the ratio of the longitudinal modulus and the shear modulus of the filament.

Through integration,

Yarn modulus,
$$\hat{E}(\alpha) = E_z$$

 $\times \left[\left(\frac{3T_0 + 1}{2dT_0} \right) + \frac{(1-d)^2}{d^3 \tan^2 \alpha} \ln \frac{(1-d)T_0 + d}{T_0} \right]$ (7)

After this approximation, only a shear-coupling effect is incorporated in the model to affect the properties of anisotropic materials and transverse coupling and secondary coupling are omitted.

Discussion of the Model

Consideration of Extreme Cases

1. The case of an untwisted yarn.

Statement: If the yarn is untwisted, the yarn modulus should be equal to the filament modulus at $\alpha = 0$.

For eq. (5), it is easy to prove that $\hat{E}(0) = E_z$. For eq. (7), at $\alpha = 0$, because tan $\alpha = 0$, and



Figure 2 Predicted twisted yarn modulus for an isotropic material.

$$\ln \frac{(1-d)T_0 + d}{T_0} = 0$$

L'Hopital's Rule needs to be used which yields

$$\hat{E}(0) = E_z \times \frac{1+d^2}{d^2}$$

For high-performance fiber yarns, d > 10, then $\hat{E}(0) \approx E_z$. For conventional fibers, eq. (7) was modified to yield the exact answer when $\alpha = 0$:

$$\begin{split} \hat{E}(\alpha) &= E_z \times \left[\left(\frac{3T_0 + 1}{2dT_0} \right) + \frac{(1 - d)^2}{d^3 \tan^2 \alpha} \\ & \ln \frac{(1 - d)T_0 + d}{T_0} \right] \Big/ \left(\frac{1 + d^2}{d^2} \right) \quad (7') \end{split}$$

2. The case of an isotropic material.

Statement: $E(\alpha) = E_z$ at all α 's for isotropic materials.

The yarn modulus was predicted by the model equations and the data are illustrated in Figure 2 for an isotropic material. It is clear that eq. (5) predicts a constant modulus for any twist angle and eq. (7'), the approximate relation, gives a deviation less than 3%. Both predict that the material properties for an isotropic material will not change with the material orientation, while the earlier models did predict a change.

Confirmation of the Model by Materials with Known Properties

To confirm the proposed equations, tests were performed to determine the dependency of modulus on twist for a material with known properties, and these data were compared with the derived equations. The longitudinal properties of a fiber are easy to measure, but the transverse and shear properties are very difficult to determine. Kevlar 49[®] is generally used in composite materials as a reinforcing fiber. Most of the material properties such as composite and matrix moduli and the fiber properties were tested. From these data, it is then possible to derive several unknown material constants through valid composite equations.

According to the composite properties of a Kevlar 49[®]/epoxy unidirectional composite, in which the volume content of the fiber, V_f , is 0.6 and the volume content of the matrix, V_m , is 0.4,²¹ E_{zc} = 76 GPa, E_{yc} = 5.5 GPa, and E_{sc} = 2.3 GPa. The matrix properties of epoxy are E_m = 3 Gpa and ν = 0.3, isotropic. According to the Rule of Mixtures, $E_{zc} = V_f E_z + V_m E_m$; then, E_z = 124.7 GPa, which is the same as that directly measured using an untwisted yarn sample.

According to the so-called Constant-Stress Rule^{22}

$$\frac{1}{E_{vc}} = \frac{V_f}{E_v} + \frac{V_m}{E_m}, \ \frac{1}{E_{sc}} = \frac{V_f}{E_s} + \frac{V_m}{G_m},$$

 $E_s = 6.9$ GPa, $E_y = 12.4$ GPa, so the anisotropic ratio d = 18.

By inserting these fiber properties into the equations, the twisted yarn behavior can be predicted. Figure 3 shows good agreement between the predicted yarn moduli by both the exact and approximate equations and the measured yarn modulus for different twist angles.

In summary, in a twisted yarn, the anisotropic ratio, which is the ratio between the axial modulus and radial shear modulus, determines how the degree of twist influences the yarn modulus, and this also is important information for fiberreinforced composites.

In this study, the stress state was simplified into a unidirectional stress acting along the yarn direction. As Allen¹² already showed, because of the cylindrical orthotropy, a simple uniaxial stress state is almost impossible. Hoop and radial stresses are induced by the action of an axial stress in any cylindrical anisotropic material to



Figure 3 Comparison of the predicted and measured yarn modulus of Kevlar 49[®] fiber.

form a complicated stress state. However, Allen's calculations on Kevlar[®] fiber indicate that the magnitudes of radial and hoop stress are less than 1% of the axial stress. Therefore, the effect of a complex stress state, as well as the coupling effect from the transverse Young's modulus and further coupling caused by off-axis angles, can be attributed to secondary importance and that is why they could be reasonably omitted in this model.

Examination of a Conventional Fiber by Our Model

Other researchers 2^{23} claimed that an equation $\hat{E}(\alpha) = E_z \cos^2 \alpha$ yields a good fit to the yarn modulus data of nylon fiber. It is worthwhile to compare the results from our model and the conventional " $\cos^2 \alpha$ " rule and further confirm our model. Ward²⁴ reported a full set of elastic constants for nylon fiber after careful measurements: $E_z = 3.45$ GPa, $E_y = 1.37$ GPa, $v_{yz} = 0.48$, and E_s = 0.61 GPa. Using these elastic constants, a predicted curve can be generated by our model. Zorowski and Murayama²⁵ reported a comparison of the experimental data and the " $\cos^2 \alpha$ " rule prediction for nylon fiber. The predictions from our model and their data are shown together in Figure 4. Both curves describe the true behavior well, because for low anisotropic ratio materials, the coupling effect from the shear modulus does



Figure 4 Comparison of the twist effect on the yarn modulus of two models for nylon fiber, $\cos^2 \alpha$ rule, and our model with the experimental data²³; lines are predicted curves and dots are experimental data.

not significantly deviate from the twisted yarn modulus from the earlier models. This might be one of the reasons why the importance of the anisotropic nature of the fiber did not draw enough attention in the past.



Figure 5 Yarn strength with the twist angle for various fibers.



Figure 6 Strain to break of a yarn with the twist angle for various fibers.

Twist Effect on Yarn Properties and Applications of the Model

The influence of the degree of twist on yarn properties, such as the yarn modulus, yarn strength, and elongation to break, was systematically studied for several high-performance fibers. The results are shown by Figures 5–7.

A general trend for the change of the yarn strength with the twist angle is that there is an optimum degree of twist for the yarn to achieve maximum strength. The magnitude of the relative strength at the same twist angle depends on the material. Kevlar 49[®] shows the largest increase in strength at the optimum twist degree, which is followed by Vectran HS[®], Kevlar 29[®], and Kevlar 149[®]. Spectra[®] and Technora[®] show only a slight increase in strength. The optimum twist angle for all the material was found to be near 7°. Generally, the elongation to break of the twisted varn increases with the twist degree. Again, the magnitude of the increment depends on the material, and Kevlar49® shows the largest increase while Spectra[®] shows almost no change.

Extensive studies on the influence of the degree of twist on strength and extensibility were done on continuous fiber yarns of several textile fibers such as acetate, nylon, viscose, and tenasco^{26–28} as well as spun yarns made from staple fiber.^{29,30} Generally, for yarns made from staple fiber, the strength shows a maximum while the extension to break increases with the degree of twist.^{29,30} For some continuous fiber yarns studied, both the strength²⁸ and the breaking extension³¹ of twisted yarns show a maximum at a certain surface twist angle. The reason for the increase in strength is likely due to an interlocking mechanism where the filaments are held together by radial forces and friction and, in effect, enables a single fiber to fail more than once. It is also possible that this transverse compression alters the stress state for the material and results in a different strength in a complex failure criterion. The prediction of the strength of twisted yarn is an interesting topic^{32–34} but will not be pursued in this article.

A mathematical curve-fitting procedure was used to resolve the data for the yarn modulus as a function of twist for the other fibers using eq. (7) (Fig. 8). The anisotropic ratio, which was unknown for these fibers, is the only fitting parameter. After the fitting, the radial shear modulus can be calculated, as the longitudinal modulus is known from the untwisted yarn measurement. The results from this type of analysis are shown in Figure 8 where the agreement is quite good. The radial shear moduli for these fibers were calculated and are listed in Table I. These data are useful for the study of corresponding fiberreinforced composites as well as for constructing the proper geometry for ropes and cables.

An extension of the application of the model and also a further step to confirm the model is to



Figure 7 Yarn modulus changes with the twist angle for various fibers; solid line is the $\cos^2 \alpha$ rule.



Figure 8 Curve-fitting results of the change of the yarn modulus with the twist degree for various fibers. In the plot, solid lines are the fitting curves and dots are the experimental data; the fitting parameter is shown in the plot.

consider a more complicated heterogeneous twisted yarn where the twisted yarn consists of two different types of fibers. The yarn modulus of mixed fibers can be predicted according to the arrangement of the different fibers inside the yarn by using the model. A different response is to be expected if the arrangements of the two fibers inside the yarn are different, since the behavior is determined by the material constants and their radial placement. A heterogeneous yarn consisting of Kevlar 149[®] and Technora[®] was examined. Two forms of the geometry as shown by Figure 9 were taken: One has Kevlar 149[®] fiber as the core and Technora[®] as the outer layer; the other has the opposite arrangement. To form a heterogeneous twisted yarn with the same twist degree distribution as of a homogeneous twisted yarn, the core material was twisted first to a certain twist degree calculated by the radius of the core and the outer layer was twisted carefully around the core layer. The yarn moduli of these two yarns at different twist angles were measured and com-



Figure 9 Illustration of a heterogeneous yarn. KevTe yarn has Kevarl 149[®] core and Technora[®] outer layer. TecKe yarn has Technora[®] core and Kevlar 14[®] outer layer.

pared to the predicted value using the anisotropic ratios determined earlier. The result is shown in Figure 10, where the experimental data and predicted values show good agreement.

CONCLUSIONS

An analysis based on composite theory was used to model the influence of twist on the yarn modulus. It is shown that the material anisotropic ratio, which is the ratio between the longitudinal modulus and the radial shear modulus, is an important factor in determining the influence of twist on the yarn modulus. For high-performance fibers, it is validated that a simple equation containing two variables, the twist angle and the material anisotropic ratio, can be used to predict the change of the yarn modulus with twist for a variety of fibers. From this analysis, it was determined that Kevlar 49[®], Kevlar 149[®], and Spectra 900[®] fibers have higher anisotropy than that of



Figure 10 Comparison of (lines) predicted data and (dots) experimental data of the twist effect on modulus of heterogeneous yarns.

Technora[®], Vectran HS[®], and Kevlar 29[®]. The calculated radial shear moduli provide useful information for the study of fiber-reinforced composites. Experimental results also show that there is an optimum twist angle of around 7° at which all the fiber yarns exhibit maximum tensile strength, but the magnitude of the increase depends on the material. The elongation to break of the twisted yarns increases with the twist degree for all of the fiber yarns investigated.

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Table I Anisotropic Ratio and Calculated Radial Shear Modulus For Different Fibers

	Vectran HS®	Technora®	Kevlar 29®	Kevlar 49®	Kevlar 149®	Spectra 900®
$d = E_z/E_s$	8.9	8.8	9.9	17.8	16	12.1
E_s	11.9	8.3	8.9	6.5	9.2	4.5

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